

Prediction of distance in hammer throwing

JESÚS DAPENA,^{1*} MARCOS GUTIÉRREZ-DÁVILA,² VÍCTOR M. SOTO² and FRANCISCO J. ROJAS²

¹Department of Kinesiology, Indiana University, Bloomington, IN 47405, USA and ²Facultad de Ciencias de la Actividad Física y del Deporte, University of Granada, Spain

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The aim of this study was to determine how much the predicted distance of a hammer throw is affected by (1) ignoring air resistance and (2) assuming that the centre of mass of the hammer coincides with the centre of the ball. Three-dimensional data from actual throws (men: 72.82 ± 7.43 m; women: 67.78 ± 4.02 m) were used to calculate the kinematic conditions of the hammer at release. A mathematical model of the hammer was then used to simulate the three-dimensional airborne motion of the hammer and to predict the distance of the throw. The distance predicted for vacuum conditions and using the ball centre to represent the hammer centre of mass was 4.30 ± 2.64 m longer than the official distance of the throw for the men and 8.82 ± 3.20 m longer for the women. Predictions using the true centre of mass of the hammer reduced the discrepancy to 2.39 ± 2.58 m for the men and 5.28 ± 2.88 m for the women. Predictions using air resistance and the true centre of mass of the hammer further reduced the discrepancy to -0.46 ± 2.63 m for the men and 1.16 ± 2.31 m for the women. Approximately half the loss of distance produced by air resistance was due to forces made on the ball and the remainder to forces made on the cable and handle. Equations were derived for calculation of the effects of air resistance and of the assumption that the centre of mass of the hammer coincides with the centre of the ball, on the distance of the throw.

Keywords: aerodynamics, biomechanics, computer simulation, hammer throw.

Introduction

The distance of a hammer throw is determined by the kinematic conditions of the hammer at release, in particular the velocity vector of the centre of mass, and by the aerodynamic forces exerted on the hammer during the flight. If the distance of a throw is estimated from the velocity and location of the centre of mass at release without taking air resistance into account, this will yield an overestimate of the distance of the throw.

There are marked discrepancies in the current literature regarding the magnitude of the effects of aerodynamic forces on the distance of hammer throws. Hubbard (1989) modelled the men's hammer ball, cable and handle, and derived an analytical solution that led him to estimate that air resistance would reduce the distance of a 90 m throw to approximately 84.3 m, a difference of 5.7 m. Soon afterwards, de Mestre (1990) developed a somewhat different analytical solution using a model that predicted smaller air

resistance forces than Hubbard's model. De Mestre did not report specific values for the effects of air resistance on hammer throwing distances, but his equations and parameters can be used to predict that air resistance would reduce the distance of a throw from 85.9 m to 84.3 m. The difference of only 1.6 m is much smaller than that predicted by Hubbard. Neither author validated the predictions against data gathered from actual throws.

It is common practice in studies of hammer throwing to assume that the velocity vector of the centre of the hammer ball at release is representative of the velocity vector of the centre of mass of the hammer. However, due to the mass of the cable and handle, the centre of mass is located slightly proximal from the centre of the ball. Because of the hammer's rotation, at the instant of release the linear velocity of the centre of the ball is larger than the linear velocity of the hammer centre of mass. Consequently, the assumption that the linear velocity of the hammer ball represents the linear velocity of the hammer centre of mass will further inflate the predicted distance of the throw.

The aims of this study were to determine (1) the effect of air resistance on the distance of the hammer

* Author to whom all correspondence should be addressed.
e-mail: dapena@indiana.edu

throw and (2) the amount of error that is introduced in the prediction of the distance based on the assumption that the centre of mass of the hammer coincides with the centre of the ball. The results were validated using data based on actual throws.

Methods

The hammer used in track and field athletics consists of a spherical metal ball with a handle attached to it through a cable (Fig. 1). Measurements taken from a standard hammer were used to produce a mathematical model. The ball was modelled as a sphere (diameter of 120 mm for the men's hammer and 102.5 mm for the women's hammer) (IAAF, 1988). The cable and handle are identical in the men's and women's hammers. The cable was modelled as three cylinders arranged in series (diameters of 6, 3 and 6 mm; lengths of 150, 687 and 150 mm, respectively). The handle was modelled as six cylinders, five of which (diameters of 8 mm; lengths of 62, 30, 117, 30 and 62 mm, respectively) formed an open pentagon closed by the sixth cylinder (diameter of 8 mm; length of 27 mm), which served as an attachment to the cable (see Fig. 1).

Volumes and areas were calculated for all elements of the model. Masses for all cable and handle elements were calculated from their respective volumes and the

known density of steel ($7.83 \text{ kg}\cdot\text{l}^{-1}$) (Byars and Snyder, 1975). Masses and distances were used to calculate the distance from the centre of the ball to the centre of mass of the hammer (26 mm for the men's hammer and 47 mm for the women's hammer) and the moment of inertia of the hammer about a transverse axis passing through the centre of mass.

The drag coefficient for the ball, C_{DB} , was estimated at 0.42, based on experimental data for smooth spheres (Schlichting, 1960) and approximate Reynolds numbers of 2.0×10^5 for men and 1.7×10^5 for women. The drag coefficients for the single cable, twined cable and handle cylinders, C_{DC} , were estimated at 1.00, 1.12 and 1.12, respectively, based on experimental data for smooth circular cylinders (Schlichting, 1960) and approximate Reynolds numbers of 5.0×10^3 , 1.0×10^4 and 1.35×10^4 , respectively.

The hammer throwing data were taken from previous research (e.g. Dapena, 1986). The data consisted of the three-dimensional coordinates of four body landmarks (two wrists and two knuckles) and of the centre of the hammer ball in 29 throws (23 by males; 6 by females) from four competitions and one training session. The throws were recorded at 50 images per second with two video or 16 mm motion-picture cameras. The positions of the five landmarks were digitized in every image of the two cameras. The non-linear transformation method (Dapena et al., 1982;

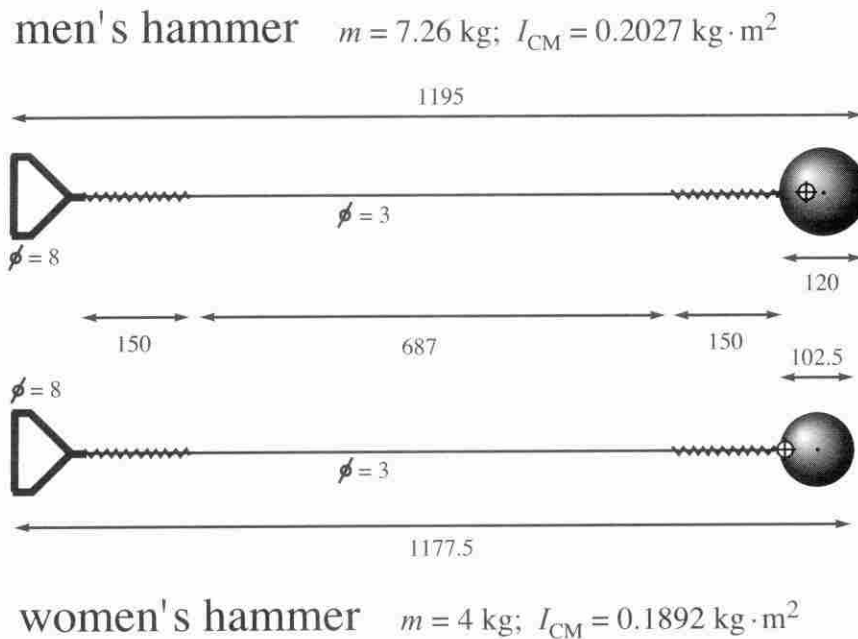


Fig. 1. Characteristics of typical men's and women's hammers. All lengths and diameters are in millimetres. The centre of mass of each hammer is indicated by the cross on the white circle. The cable is attached to the handle and ball through loops and twines around itself for about 150 mm at both ends. m = mass; I_{CM} = the moment of inertia of the hammer about a transverse axis passing through the centre of mass.

Dapena, 1985) was used to compute three-dimensional coordinates for three of the sessions; the direct linear transformation method (Abdel-Aziz and Karara, 1971; Walton, 1981) was used for the remaining two sessions. Quintic spline functions (Wood and Jennings, 1979) were fitted to the raw three-dimensional location values of the five landmarks to compute their instantaneous locations and velocities at the approximate instant of release, which generally occurred between frames. To prevent the possible introduction of systematic errors, the spline functions were applied with zero smoothing.

The angular velocity vector of the hammer at the instant of release was computed as the angular velocity of the line joining the centre of the hammer ball and the average position of the wrists and knuckles at the instant of release. The linear velocity vector of the centre of the hammer ball, the angular velocity vector of the hammer, and the location vector of the hammer centre of mass relative to the centre of the ball at the instant of release were used to calculate the linear velocity vector of the hammer centre of mass at the instant of release.

The conditions of the hammer at release were used to produce three separate kinds of simulations for the flight of the hammer. For the first simulation, we assumed that the location and velocity vectors of the ball centre at release represented the location and velocity vectors of the hammer centre of mass. Their values were input to the standard equations of airborne motion in a vacuum to estimate the hypothetical distance of the throw in the absence of aerodynamic forces. The distance predicted by this simulation was denoted by R_{BV} .

The second simulation was similar to the first, but used the true location and velocity of the hammer centre of mass at release. The distance predicted by this simulation was denoted by R_{GV} .

The third simulation considered the true location and velocity of the centre of mass of the hammer at release, as well as the hammer's three-dimensional orientation and angular velocity vector at release, and the aerodynamic forces exerted on the hammer ball, cable and handle during the airborne phase. For this simulation, each twined cable section was further subdivided into two equal subsections, and the simple cable section was subdivided into eight subsections of equal length.

The drag force exerted by the air on the ball was calculated using the following equation:

$$F_{DB} = 0.5 \rho C_{DB} A_B v^2 \quad (1)$$

where ρ is the estimated density of air at 25°C (1.175 kg·m⁻³) (see Dapena and Feltner, 1987), A_B is the projected area of the ball and v is the magnitude of the instantaneous velocity of the still air relative to the

ball. Vector F_{DB} pointed in the direction of the airflow relative to the ball.

The force exerted by the air on each cable and handle cylinder was calculated using the following equation:

$$F_C = 0.5 \rho C_{DC} A_C v^2 \quad (2)$$

where A_C is the projected lateral area of the cylinder and v is the magnitude of the component of the instantaneous relative airflow in the direction perpendicular to the longitudinal axis of the cylinder. Vector F_C was perpendicular to the longitudinal axis of the cylinder and was contained in the plane defined by the longitudinal axis and the relative airflow.

The torque vector T exerted by each aerodynamic force F relative to the centre of mass of the hammer was calculated using the equation

$$T = r \times F \quad (3)$$

where r is the vector pointing from the hammer centre of mass to the centre of the hammer element on which the aerodynamic force was exerted.

The aerodynamic forces and the weight of the hammer were added to calculate the resultant force vector ΣF exerted on the hammer; the torques exerted by the aerodynamic forces were added to calculate the resultant torque vector ΣT exerted on the hammer.

The instantaneous linear and angular accelerations of the hammer (a and α , respectively) were calculated using the following equations:

$$a = \Sigma F/m \quad (4)$$

$$\alpha = \Sigma T/I_{CM} \quad (5)$$

where m is the total mass of the hammer (7.26 kg for the men's hammer and 4 kg for the women's hammer) and I_{CM} is the moment of inertia of the hammer about a transverse axis passing through the centre of mass (0.2027 kg·m² for the men's hammer and 0.1892 kg·m² for the women's hammer).

We assumed that the linear and angular accelerations remained constant for a short time interval (Δt) after release. This permitted the calculation of estimated values for the location and linear velocity of the hammer centre of mass, and orientation and angular velocity of the hammer, at the end of the interval. In turn, the new linear velocity, orientation and angular velocity values were used to calculate new instantaneous linear and angular accelerations at the end of the interval. These updated accelerations were then assumed to remain constant for a subsequent Δt interval. The process was repeated until the centre of mass of the hammer

reached ground level. The distance of the throw predicted by this simulation was denoted by R_{GA} . Values of Δt between 1.0 and 0.001 s were tested. The decrease in the value of Δt from 0.01 s to 0.001 s was found to change the predicted distance of a 74 m throw by only 0.005 m. The value $\Delta t=0.01$ s was thus established as a good compromise between accuracy and computer time expenditure and was adopted for all simulations.

Since hammer throws are measured to the nearest edge of the mark made on the ground by the ball, in all simulated throws the predicted distance of the throw (R_{BV} , R_{GV} and R_{GA}) was reduced by a correction distance $\Delta R = d_B / (2 \cdot \sin \theta_L)$, where d_B is the diameter of the hammer ball and θ_L is the angle between the horizontal plane and the velocity vector of the hammer at the instant of landing.

Results

Table 1 shows the official distances of the throws and the distances predicted by the three different computer simulations. The distance predicted for vacuum conditions using the ball centre to represent the hammer centre of mass (R_{BV}) was 4.30 ± 2.64 m longer than the official distance of the throw for the men and 8.82 ± 3.20 m longer for the women (see Figs 2a,b). Predictions using the true centre of mass of the hammer and vacuum conditions (R_{GV}) reduced the discrepancy regarding the official distance of the throw (differences of 2.39 ± 2.58 m for the men and 5.28 ± 2.88 m for the women). Predictions using the true centre of mass of the hammer and air resistance (R_{GA}) reduced still further the discrepancy regarding the official distance of the throw: differences of -0.46 ± 2.63 m for the men and 1.16 ± 2.31 m for the women (see Figs 2c,d).

Figure 3a,b shows the relationship between the effect of air resistance on the distance of the throw

($R_{GV} - R_{GA}$) and the official distance of the throw (R). Theoretical considerations based on Lichtenberg and Wills (1978) indicated that this relationship should be approximately quadratic. Accordingly, a quadratic equation passing through the origin was fitted to the data points.

Figure 3c,d shows the relationship between the overestimate in the predicted distance of the throw based on the assumption that the centre of the ball represents the centre of mass of the hammer ($R_{BV} - R_{GV}$) and the official distance of the throw (R). Theoretical considerations indicated that this relationship should be approximately linear. Accordingly, a straight line passing through the origin was fitted to the data points.

The official distance of a throw can be input into the equations shown in Fig. 3 to estimate the distance that would have been achieved if the throw had taken place in a vacuum and if the kinematic conditions of the centre of the hammer ball at release had represented the kinematic conditions of the centre of mass.

Discussion and sensitivity tests

Figures 2a and 2b and the difference between the mean values of R_{BV} and R in Table 1 indicate that neglecting the effect of air resistance and assuming that the centre of the hammer ball represents the centre of mass of the hammer will lead to greatly inflated predictions of the distances of hammer throws (R_{BV}). The validity of the model developed in this study was supported by the small sizes of the residual discrepancies between the official distances (R) and the distances predicted by the full model in men's and women's throws (R_{GA}) (see Table 1 and Figs 2c,d).

The remaining differences between the official distances (R) and the distances predicted by the model (R_{GA}) were due only in part to imperfections in the model; they were also due to random and systematic errors in the calculation of release conditions and to the unknown wind conditions during the recording sessions. Random errors in the digitization of the hammer ball, knuckles and wrists produced random errors in the estimates of release conditions for each trial and, therefore, random errors in the model's predictions of distances for individual throws. Lens distortion and random errors in the digitization of the control object used for three-dimensional analysis produced systematic errors in the estimates of release conditions for each separate recording session. Because of the systematic errors, the trials of any particular session may show a consistent bias that overpredicts or underpredicts the distances of the throws. Wind conditions were not monitored during the recording sessions and, therefore,

Table 1. Distances of the throws (mean \pm s)

	Distance thrown (m)	
	Men	Women
R	72.82 ± 7.43	67.78 ± 4.02
R_{BV}	77.12 ± 7.13	76.60 ± 6.64
R_{GV}	75.21 ± 6.95	73.06 ± 6.31
R_{GA}	72.36 ± 6.42	68.94 ± 5.64

Note: R = official distance; R_{BV} = predicted distance in a vacuum, using the ball centre to represent the hammer centre of mass; R_{GV} = predicted distance in a vacuum, using the true centre of mass; R_{GA} = predicted distance in air, using the true centre of mass.

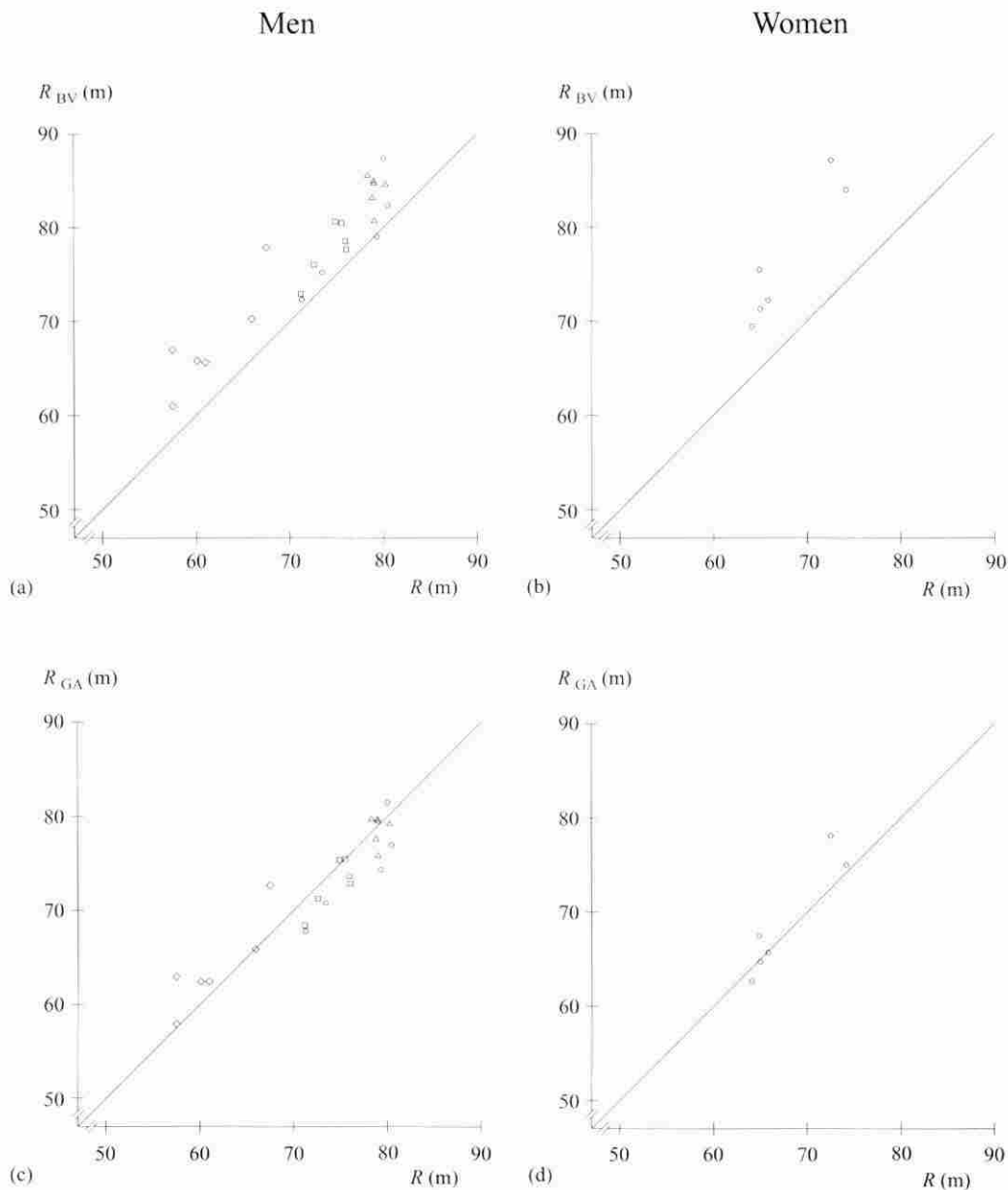


Fig. 2. Distances predicted for vacuum conditions with the ball centre representing the centre of mass of the hammer (R_{BV}) (a,b), and for air resistance conditions using the true centre of mass of the hammer (R_{GA}) (c,d), versus the official distances of the throws (R). The diagonal lines are identity lines. The various symbols represent different recording sessions.

the model was run with the assumption of zero wind for all predictions. Prevailing headwinds or tailwinds during any given recording session will have reduced or increased, respectively, the measured distances of the throws in that session in relation to the distance predicted by the model. Owing to all these sources of error, it is not surprising that there were discrepancies for individual trials or recording sessions.

Figures 3a and 3b show that the effect of air resistance is smaller in the men’s hammer than in the women’s hammer. This is because mass increases in proportion to the cube of linear dimensions, while

frontal area – and, therefore, air resistance force – increases in proportion to the square of linear dimensions. Thus, the increase in the air resistance force from the women’s hammer to the men’s hammer is smaller than the increase in the mass of the hammer. Consequently, the ratio of air resistance to mass is smaller for the men’s hammer, and so is the effect of air resistance on the distance of the throw.

Figures 3c and 3d show that the assumption that the centre of the ball represents the centre of mass of the hammer has less of an effect on the distance of the throw in the men’s hammer than in the women’s

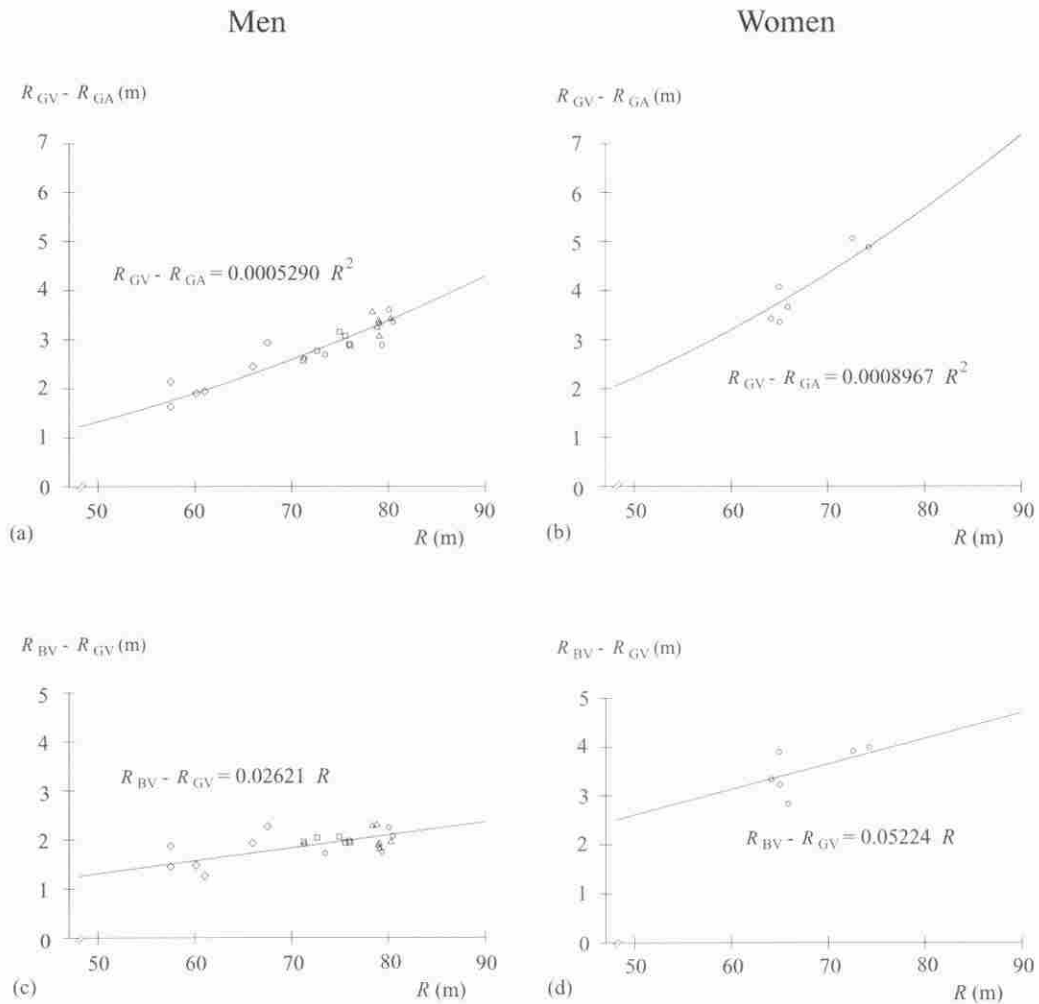


Fig. 3. Effect of air resistance on the distance of the throw ($R_{GV} - R_{GA}$) and overestimation in the predicted distance of the throw based on the assumption that the centre of the ball represents the centre of mass of the hammer ($R_{BV} - R_{GV}$), both as functions of the official distance of the throw (R).

hammer. This is because the ball has a larger mass in the men's hammer than in the women's hammer, while the cable and handle are the same. Consequently, the distance from the hammer centre of mass to the centre of the ball is smaller in the men's hammer, as are the overestimates of the hammer velocity at release and of the predicted distance of the throw.

The Reynolds numbers for the men's and women's hammer balls are about 2.0×10^5 and 1.7×10^5 , respectively. The relationship between the drag coefficient C_D and the Reynolds number is reasonably well known for smooth spheres (Schlichting, 1960; Hoerner, 1965; Achenbach, 1974). The drag coefficient has a fairly stable value (0.4–0.5) for Reynolds numbers between approximately 1×10^3 and 2.5×10^5 . This is followed by a 'critical' region in which it drops steeply to a value of about 0.09 at a Reynolds number of 4×10^5 . Given the Reynolds numbers of the men's and women's hammer balls, they might be viewed as being

clearly in the subcritical range. However, surface roughness is a confounding issue, since it tends to reduce the critical Reynolds number (Hoerner, 1965; Achenbach, 1974). The roughness of the hammer ball's surface is not known exactly; it is unclear, therefore, whether the hammer ball is above or below the critical Reynolds number. De Mestre (1990) assumed that the men's hammer ball is above the critical Reynolds number during throwing. Accordingly, he proposed a rather low drag coefficient for it ($C_D \sim 0.34$, inferred from other values reported by de Mestre, 1990). Hoerner (1965) and Achenbach (1974) reported rather large drag coefficients of about 0.50 for spheres in the subcritical region, while Schlichting (1960) reported a lower value for the same region. Schlichting's drag coefficient of 0.42 was adopted in the present study because it lay between the large subcritical values reported by Hoerner (1965) and Achenbach (1974) for smooth spheres ($C_D \sim 0.50$) and the lower value that de

Mestre (1990) proposed for the men's hammer ball ($C_D = 0.34$). A sensitivity test was carried out to assess the effect of this choice on the predictions of the model.

Simulations using the full model with drag coefficients of 0.50 and 0.34 for the hammer ball in a mid-range throw with the men's hammer (official distance = 72.62 m) produced a 0.27 m decrease and a 0.27 m increase, respectively, in the distance of the throw in relation to the result produced with the standard drag coefficient of the model ($C_D = 0.42$). These differences amounted to only 10% of the total effect of air resistance in this throw (2.77 m). A second sensitivity test was carried out to establish the reason for such a small effect.

The same throw was simulated using various air resistance conditions for the ball and for the cable and handle, and the following reductions were found in the distance of the throw (relative to the distance produced without any air resistance): simulation using air resistance only on the ball, 1.53 m; air resistance only on the cable and handle, 1.31 m; air resistance on ball, cable and handle, 2.77 m. The effects were almost additive. They showed that the aerodynamic force exerted on the cable and handle accounts for approximately 46% of the total effect of air resistance on the distance of the throw for the men, leaving the remaining 54% for the ball. The difference between Schlichting's drag coefficient of 0.42 and the other potential drag coefficients of 0.50 and 0.34 (0.08 in either direction) represents a 19% difference in the air resistance force exerted on the ball. Since 19% of 54% is 10% of the total, this fits quite well with the value reported in the previous paragraph. It indicates that a given percent change in the air resistance force produces a similar percent change in the air-induced reduction in the distance of the throw.

Similar tests were carried out for a mid-range throw with the women's hammer (official distance = 65.86 m). Simulations using drag coefficients of 0.50 and 0.34 for the hammer ball produced a 0.30 m decrease and a 0.31 m increase, respectively, in the distance of the throw in relation to the result produced with the standard drag coefficient of the model. These differences amounted to 8% of the total air resistance effect in this throw (3.70 m), slightly less than for the men. The reason for this was that the relative effects of the aerodynamic forces exerted on the ball and on the cable and handle were reversed with respect to the men: the forces exerted on the cable and handle accounted for approximately 54% of the total effect of air resistance on the distance of the throw for the women, leaving only 46% for the ball (decreases in distance due to effects of forces: on ball, 1.77 m; on cable and handle, 2.05 m; on ball, cable and handle, 3.70 m).

The critical Reynolds number for the drag coefficient of cylinders is about 2×10^5 (Schlichting, 1960; Hoerner, 1965). This is much larger than the Reynolds numbers of the cylinders that make up the hammer's cable and handle (5.0×10^3 to 1.35×10^4). The hammer model's cylinders are clearly in the subcritical region, regardless of any possible roughness (see Hoerner, 1965), and their drag coefficients are not in doubt.

Hubbard (1989) modified an equation developed by Lichtenberg and Wills (1978) and used it to estimate the effect of air resistance on the distance of a hammer throw. He predicted that air resistance would reduce the distance of a 90 m vacuum throw by about 5.7 m to produce an 84.3 m throw. In contrast, the equation that estimates the effect of air resistance in men's throws according to our model ($R_{GV} - R_{GA} = 0.0005290 R^2$) predicts that a throw measured officially at 84.3 m would have been shortened 3.8 m (from a vacuum distance of 88.1 m) by air resistance. Therefore, Hubbard's prediction of the effect of air resistance was about 50% larger than the prediction of our model.

De Mestre (1990) did not report specific values for the effects of air resistance on hammer throwing distances. However, his equations can be used to make predictions. We input an initial trajectory angle of 42° , a release height of 1.40 m and a wide variety of release velocities into de Mestre's equation (8.1) to estimate throw distances in the presence of air. His equation (8.1) requires a value for the expression $\varepsilon = 0.5 \rho C_D A v_0^2 / mg$. A value of 0.02 was assigned by de Mestre to this expression for a hammer with an initial velocity v_0 of $25 \text{ m}\cdot\text{s}^{-1}$. For each throw, we adjusted the value of the expression in proportion to the square of the initial velocity, since the other elements in the expression should remain constant. By trial and error, we found that an initial velocity v_0 of $28.84 \text{ m}\cdot\text{s}^{-1}$ would be needed to produce a throw of 84.3 m. The conditions of this throw at release were subsequently used to calculate the distance that it would have reached in a vacuum. The result was 85.9 m, which would imply an air resistance effect of 1.6 m on the distance of the throw. This was about 60% smaller than the 3.8 m effect predicted by our model.

Although the predictions of our simulation model fit quite well with the results of actual throws, the predictions of the effects of air resistance by Hubbard's and de Mestre's models were larger and smaller, respectively, than those of our model. This suggests that Hubbard's and de Mestre's models overestimate and underestimate, respectively, the effect of air resistance on the distance of hammer throws.

Conclusions

We have shown that there is a large difference between the measured distance of a hammer throw and the distance predicted from the kinematic conditions at release if the effect of air resistance is not taken into account and if the centre of the ball is assumed to represent the centre of mass of the hammer. Our model allowed us to break down the difference into two separate components: air resistance and the inaccurate identification of the ball centre with the centre of mass of the hammer. The model also showed that the effects of air resistance are shared in approximately equal amounts by the ball and by the cable and handle. The results suggested that Hubbard's and de Mestre's models overestimate and underestimate, respectively, the effect of air resistance on the distances of hammer throws. From a practical standpoint, the predictor equations obtained in this study will allow researchers to reconcile the predictions of hammer throw distances made from the conditions at release with the distances measured by the officials.

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