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Influence of environmental factors on shot put and hammer throw range

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Abstract

On the rotating Earth, in addition to the Newtonian gravitational force, two additional relevant inertial forces are induced, the centrifugal and Coriolis forces. Using computer modelling for typical release heights and optimal release angles, we compare the influence of Earth rotation on the range of the male hammer throw and shot put with that of air resistance, wind, air pressure and temperature, altitude and ground obliquity. Practical correction maps are presented, by which the ranges achieved at different latitudes and/or with different release directions can be corrected by a term involving the effect of Earth rotation. Our main conclusion and suggestion is that the normal variations of certain environmental factors can be substantially larger than the smallest increases in the world records as acknowledged by the International Amateur Athletic Federation and, therefore, perhaps these should be accounted for in a normalization and adjustment of the world records to some reference conditions. Although this suggestion has certainly been made before, the comprehensiveness of our study makes it even more compelling. Our numerical calculations contribute to the comprehensive understanding and tabulation of these effects, which is largely lacking today.

Keywords: Hammer throw; Shot put; Range correction; Environmental factors; Earth rotation

1. Introduction

Nowadays, differences ΔL_r of the throw distances L_r of consecutive world records are only a few centimetres in both the female and male shot put, and a few decimetres in the male hammer throw (Table 1). Thus the question arises whether ΔL_r have become comparable to the change ΔL of range L due to usual variations of environmental factors. For the sake of fair comparison of sport performances, certain rules must be followed in international athletic competitions. For example, a 2 m/s tail wind speed limit for record purposes is applied by the International Amateur Athletic Federation (IAAF) to sprint races and horizontal jumps. Wind also appreciably influences the range of the throwing events.

On the rotating Earth, due to the centrifugal and Coriolis (inertial) forces, the latitude and the release direction also influence the motion. A gross analytical estimation shows that under certain circumstances the change of the drag-free range of the two most inertially dominated throws, the shot put and hammer throw due to variations in latitude can be greater than ΔL_r. A further question is whether this change of the throw distance is not overwhelmed by the effect of natural fluctuations of other environmental factors, such as altitude, wind, or variation of the air density due to a change in air pressure and temperature, for instance.

Although numerous excellent comprehensive and detailed analytical and computer studies exist in the literature on the mathematics and mechanics of the shot put and hammer throw (e.g. Garfoot, 1968; Tutevich, 1969; Lichtenberg and Wills, 1978; Zatsiorsky et al., 1981; Hay, 1985; Hubbard, 1989; de Mestre, 1990; Zatsiorsky, 1990; Hubbard et al., 2001), detailed numerical computations of the effect of Earth rotation on range have never been reported previously. Heiskanen (1955) gave a gross analytical estimation for the effect of the centrifugal force on the drag-free range of the four throwing events. We do not know of any published calculations of the effect of the Coriolis force on range.

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Table 1
Progression of the ten best consecutive world records (L_r), their differences (ΔL_r) and the years in which they were achieved for the male hammer throw and shot put (Megede and Hymans, 1991). The present world records have not fallen since 1986 and 1990

Male hammer throw			Male shot put		
L_r (cm)	ΔL_r (cm)	Year	L_r (cm)	ΔL_r (cm)	Year
8038	6	1980	2200	15	1976
8046	8	1980	2215	15	1978
8064	18	1980	2222	7	1983
8166	102	1980	2262	40	1985
8180	14	1980	2264	2	1986
8389	209	1982	2272	8	1987
8414	25	1983	2284	12	1987
8634	220	1984	2291	7	1987
8666	32	1986	2306	15	1988
8674	8	1986	2312	6	1990

To fill this gap, we performed an in-depth computer modelling (Mizera, 1999) including, in addition to the effect of Earth rotation, all other relevant environmental factors affecting the range of the male shot put and hammer throw. We computed and compared the effect of variation in air resistance (drag), wind speed, air pressure and temperature, altitude, latitude, release direction and ground obliquity on throw distance for typical release (initial) conditions. Practical correction maps were computed, by which the ranges achieved at different latitudes and/or with different release directions can be correctly compared with each other in such a way, that the throw distances are corrected by a term involving the influence of Earth rotation. We show that the magnitude of change in range due to changes in latitude is comparable to or even larger than those of other environmental factors. Analysing the evolution of the world records of shot put and hammer throw, we demonstrate that the time has come to take into account the effect of certain environmental variables on range. In this work the word "environmental" is used to include all factors examined, even Coriolis and centrifugal force effects which depend abstractly on latitude and direction in much the same way as air density does on altitude. To our knowledge, this is the first precise analysis of the effect of Earth rotation on throw distance in comparison with the influences of other environmental factors.

In this work we deal only with the male shot put and hammer throw. The corresponding female throwing events would require a separate study, because the masses and dimensions of the women's implements are different. In the discus and javelin throws, due to atmospheric fluctuations (wind and change of air density), the effect of aerodynamics on range may overwhelm the influence of all other environmental factors.

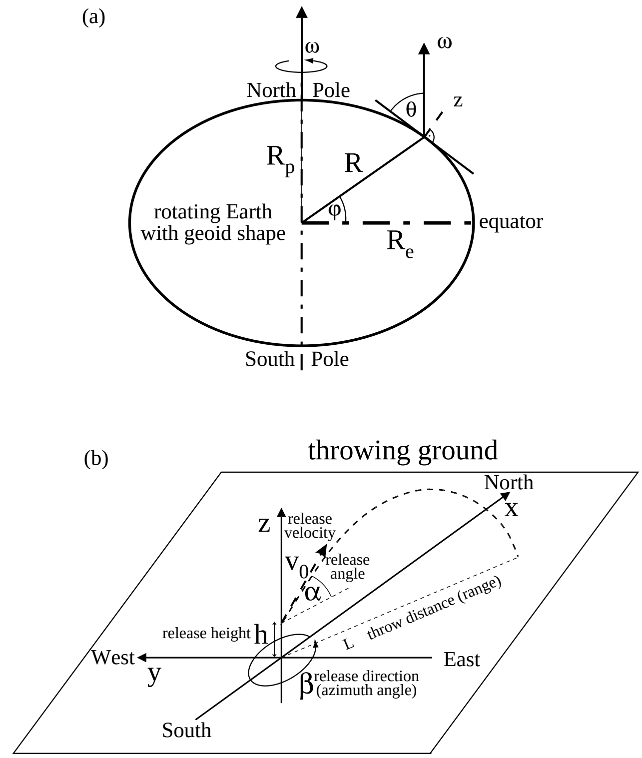

Fig. 1. (A) Geometry and variable definitions of the rotating Earth geoid. (B) Definition of computer simulation variables. Terminology and geometry of the release (v_0, h, α, β) and impact (L) parameters.

2. Computer modelling of the flight trajectory of the shot and hammer

Consider a Cartesian system of coordinates x–y–z fixed to the Earth surface (Fig. 1B). The x- and y-axis point northward and westward, respectively, and the z-axis points vertically upwards (perpendicular to the geoid surface, Fig. 1A). According to Landau and Lifschitz (1974), in this rotating reference system the motion equation of a thrown shot or hammer is

$$m\frac{d^2\underline{r}}{dt^2} = m\underline{g}(\varphi, H) + 2m\frac{d\underline{r}}{dt} \times \underline{\omega} - \frac{kA\rho(p,T)}{2}\left(\frac{d\underline{r}}{dt} - \underline{W}\right)^2, \tag{1}$$

where $\times$ represents the vector cross product. The variables and formulae used in the computer simulation are the following (Fig. 1):

(i) $\underline{r} = (x, y, z)$: position vector of the implement.

(ii) φ: latitude (positive or negative on the northern or southern hemisphere, respectively).

(iii) H: altitude determining the gravitational acceleration g and the air pressure p in the following way:

$$g(H) = g_0\left(1 - \frac{2H}{R}\right), \quad p = p_0\, e^{-0.000123\ \mathrm{m}^{-1}\ H},$$
$$g_0 = 9.81\ \mathrm{m/s^2}, \quad R = 6.368 \times 10^6\ \mathrm{m},$$
$$p_0 = 101\,325\ \mathrm{Pa}, \tag{2}$$

where R is the average of the Earth radius.

(iv) $\underline{\omega}$: Earth angular velocity vector, with amplitude $\omega = 7.27 \times 10^{-5}\ \mathrm{s}^{-1}$, and direction parallel to the axis of rotation. Its Cartesian coordinates would be $\omega_x = \omega \cos\varphi$, $\omega_y = 0$, $\omega_z = \omega \sin\varphi$ if the Earth possessed a spherical shape. However, the Earth has a flattened geoid shape, approximately an ellipsoid with rotational symmetry (Fig. 1A). Due to this flattened shape, apart from the equator and the poles, at latitude φ the angle between $\underline{\omega}$ and the Earth surface is $\theta(\neq\varphi)$. If R_e and R_p are the Earth equatorial and polar radii, respectively, θ and the coordinates of $\underline{\omega}$ can be expressed as (Fig. 1A)

$$\tan\theta = \frac{R_e^2}{R_p^2}\tan\varphi, \quad \omega_x = \omega\cos\theta, \quad \omega_y = 0,$$
$$\omega_z = \omega\sin\theta, \tag{3}$$

where $R_e = 6.378 \times 10^6$ m, $R_p = 6.357 \times 10^6$ m.

(v) $m\underline{g}$: gravitational force, where the "apparent" gravitational acceleration (Caputo, 1967)

$$\underline{g} = (g_x, g_y, g_z) = [0, 0, -g(\varphi)],$$

$$g(\varphi) = 9.78049\,(1 + 0.0052884\sin^2\varphi - 0.0000059\sin^2 2\varphi)\ \mathrm{m/s^2} \tag{4}$$

involves both the geoid Earth shape (latitude dependent Earth radius) and the Earth rotation (latitude dependent vertical component of the centrifugal acceleration).

(vi) $\underline{W}$: wind velocity vector.

(vii) $\rho = 1.23\ \mathrm{kg/m^3}$: air density under normal conditions (temperature $T_0 = 293$ K and pressure $p_0 = 101\,325$ Pa). It depends on p and T as follows:

$$\rho(p, T) = \frac{p}{QT}, \quad Q = 287.05\ \mathrm{J/kg\ K}. \tag{5}$$

(viii) β: release direction (or azimuth angle) measured counter-clockwise from North.

(ix) α: release angle measured from the horizontal.

(x) h: release height measured from the ground. Although it is thrower-specific, we assumed $h_h = 1.8$ m for the hammer throw and $h_s = 2.25$ m for the shot put (Hubbard, 1989).

(xi) v_0: release velocity.

(xii) $m = 7.26$ kg: mass of the male shot and hammer.

(xiii) t: time measured from the moment of release.

(xiv) $2\,\mathrm{m}\frac{\mathrm{d}\underline{r}}{\mathrm{d}t} \times \underline{\omega}$: Coriolis force.

(xv) $\frac{k\rho A}{2}(\frac{\mathrm{d}\underline{r}}{\mathrm{d}t} - \underline{W})^2$: drag force due to air resistance, where A is the projected area of the shot ($A_s = 0.0095\ \mathrm{m^2}$) and hammer ($A_h = 0.0138\ \mathrm{m^2}$), k is the drag coefficient of the shot ($k_s = 0.47$) and hammer ($k_h = 0.70$). The exact value of k_h varies along the trajectory and depends on the attitude of the wire and handle relative to the flight path. Furthermore, the handle moves erratically around the head during flight, the consequence of which is that its drag varies too. $k_h = 0.70$ is an averaged effective value (Hubbard, 1989). In the case of the typical average velocities of 15 and 30 m/s of the shot and hammer during flight, respectively (Hubbard, 1989), the air resistance is proportional to the square of the relative velocity vector $\mathrm{d}\underline{r}/\mathrm{d}t - \underline{W}$.

Motion equation (1) involves all relevant environmental (physical and meteorological) factors determining the trajectory of a thrown shot or hammer. We solved (1) by means of Runge–Kutta numerical integration of fourth order.

3. Results

In the case of the male world-record hammer throw (achieved in 1986) the initial conditions of the computer simulation were (Fig. 1B): $h = 1.8$ m, $\alpha = 44°$ (ballistically optimal release angle) and $v_0 = 29.28$ m/s corresponding to a throw with $L = 86.74$ m. For the male world-record shot put (achieved in 1990) the release conditions were: $h = 2.25$ m, $\alpha = 37°$ and $v_0 = 14.5$ m/s corresponding to a throw with $L = 23.12$ m. In simulations studying the influence of Earth rotation on range, the standard values of environmental parameters were: air temperature $T_0 = 293\ \mathrm{K} = 20°\mathrm{C}$, air pressure $p_0 = 101\,325$ Pa, altitude $H = 0$ m, ground obliquity $o = 0$. As reference city, to which all records are normalized, we chose Athens. This was an obvious choice, because both the next (in 2004) and first Olympic Games will be/were held there, and because it is at a reasonably central latitude. Thus, as reference latitude we chose the latitude of Athens $\varphi = 38°$, and as reference release direction we used the northern direction $\beta = 0°$ (Fig. 1B).

3.1. Range versus latitude

Fig. 2 shows the throw distance of the world-record hammer throw as a function of latitude for four release directions. At a given latitude, due to the Coriolis force, the difference of the ranges for different release directions is maximal at the equator and gradually decreases to zero toward the poles. At a given release direction the range gradually increases from the poles toward the equator because of the centrifugal force and increasing Earth radius, which have much greater influences on range than the Coriolis force.

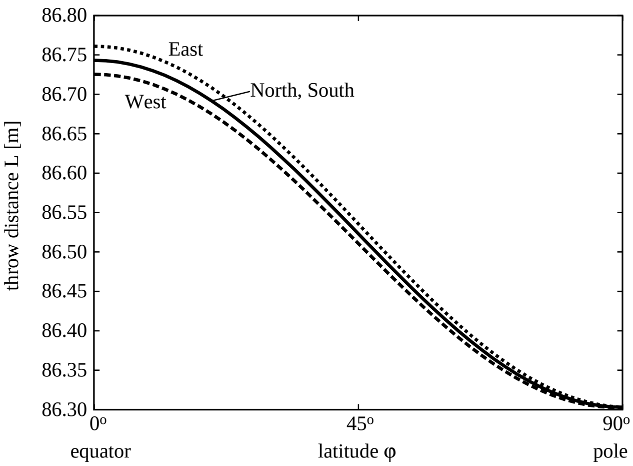

Fig. 2. Male world-record hammer throw distance L as a function of latitude φ for four release directions (East, West, North, South) when $T = 20°C$, $p = 101\,325$ Pa, $H = 0$ m, $o = 0$, $W = 0$ m/s.

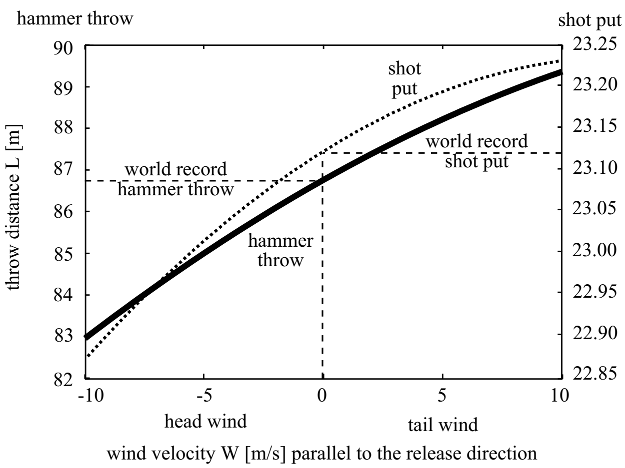

Fig. 3. Male world-record hammer throw distance L as a function of wind velocity W parallel to the release direction for Athens' latitude $\varphi = 38°$, $T = 20°C$, $p = 101\,325$ Pa, $H = 0$ m, $o = 0$.

3.2. Influence of wind

Fig. 3 plots the influence of wind (parallel to the release direction) on hammer throw and shot put range at the reference latitude of Athens $\varphi = 38°$, as a function of the component of the wind velocity vector parallel to the release direction. The effect of the wind speed on range is non-linear. The increase in range from a tail wind is smaller than the decrease from a head wind of the same speed.

3.3. Range versus altitude

Table 2 shows how the throw distance of the hammer throw and shot put increases with altitude H ranging between 0 and 1000 m. We chose this altitude range, because the majority of the world's human population

Table 2
Male world-record hammer and shot range L as a function of altitude H for Athens' latitude $\varphi = 38°$, $o = 0$, $W = 0$ m/s, $T = 20°C$. The dependence is approximately linear

Altitude H (m)	0	200	400	600	800	1000
Hammer throw L (m)	86.74	86.86	86.97	87.08	87.19	87.29
Shot put L (m)	23.120	23.125	23.129	23.133	23.137	23.142

Table 3
The effect of vertical level difference Δz along throw distance on the male world-record hammer and shot range L for Athens' latitude $\varphi = 38°$, $T = 20°C$, $p = 101\,325$ Pa, $H = 0$ m, $W = 0$ m/s. The dependence is approximately linear

Δz (cm)	−20	−10	0	10	20
Hammer throw L (m)	86.93	86.84	86.74	86.65	86.55
Shot put L (m)	23.33	23.22	23.12	23.02	22.91

lives at altitudes <1000 m, and every Summer Olympic Games venue in the last 100 years has occurred at altitude <100 m with only three exceptions (Munich, $H = 320$ m; Atlanta, $H = 450$ m; and Mexico City, $H = 2200$ m). The increase of range with altitude is due to the decrease of the gravitational acceleration g and the air density ρ, the latter as a result of the decrease in air pressure p.

3.4. Influence of ground obliquity

Table 3 shows the influence of ground obliquity on range of the hammer throw and shot put. If the vertical level difference of the throwing ground is Δz along the throw distance L, ground obliquity is defined as $o = \Delta z/L$. The greater the level difference, the more the range differs on an oblique ground from that on a horizontal ground. The maximal obliquity permitted by the IAAF along the range is $o_{\max} = \pm 0.001$. The maximal permitted vertical level difference of the ground is about ± 8.7 and ± 2.3 cm for the hammer throw ($L = 86.74$ m) and shot put ($L = 23.12$ m), respectively.

3.5. Influence of air pressure and temperature

Tables 4 and 5 show the influence of air pressure p and temperature T on the hammer throw and shot put distance. We used $\Delta p = \pm 2$ kPa and $\Delta T = \pm 20°C$ for the maximal size of reasonable air pressure and temperature perturbations, respectively, because generally, larger changes do not occur under normal meteorological conditions.

Table 4
Male world-record hammer and shot range L as a function of air pressure p for Athens' latitude $\varphi = 38°$, $T = 20°C$, $H = 0$ m, $o = 0$, $W = 0$ m/s. The dependence is approximately linear

Air pressure p (kPa)	$p_0 - 2$	$p_0 - 1$	$p_0 = 101\,325$	$p_0 + 1$	$p_0 + 2$
Hammer throw L (m)	86.83	86.78	86.74	86.70	86.66
Shot put L (m)	23.122	23.121	23.120	23.119	23.118

Table 5
Male world-record hammer and shot range L as a function of air temperature T for Athens' latitude $\varphi = 38°$, $p = 101\,325$ Pa, $H = 0$ m, $o = 0$, $W = 0$ m/s. The dependence is approximately linear

Air temperature T (°C)	10	15	20	25	30	35
Hammer throw L (m)	86.57	86.66	86.74	86.83	86.91	87.00
Shot put L (m)	23.115	23.118	23.120	23.123	23.125	23.128

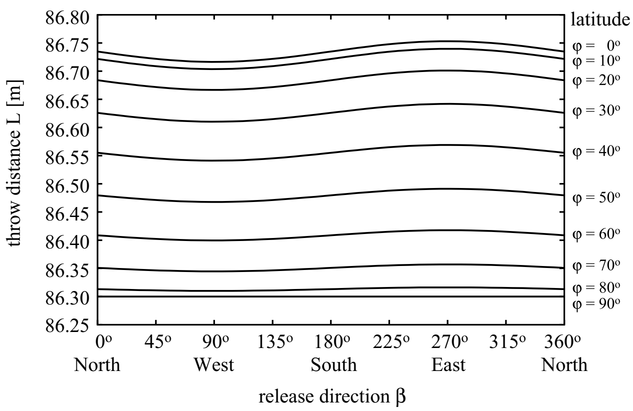

Fig. 4. Male world-record hammer throw distance L as a function of release direction β for Athens' latitude $\varphi = 38°$, $T = 20°C$, $p = 101\,325$ Pa, $H = 0$ m, $o = 0$, $W = 0$ m/s.

3.6. Range versus release direction

Fig. 4 plots the hammer throw distance as a function of release direction for ten latitudes. At a given latitude, throw distance changes approximately sinusoidally with release direction.

In a stadium, the shot and hammer must hit the ground within a 40° sector. Fig. 5 demonstrates for reference latitude $\varphi = 38°$ that, due to the Coriolis force, there is a small difference between the throw distances for the left and right border of the sector. This difference depends approximately sinusoidally on the direction of the sector centre line. If the centre line points northward or southward, the difference is maximal (about 1 cm for the hammer throw). Because the Coriolis force increases with decreasing latitude, this difference is somewhat greater at the equator ($\varphi = 0°$) than in Athens ($\varphi = 38°$), and can reach a value of about 1.5 cm. In the case of shot put this difference is <2 mm, which can be neglected.

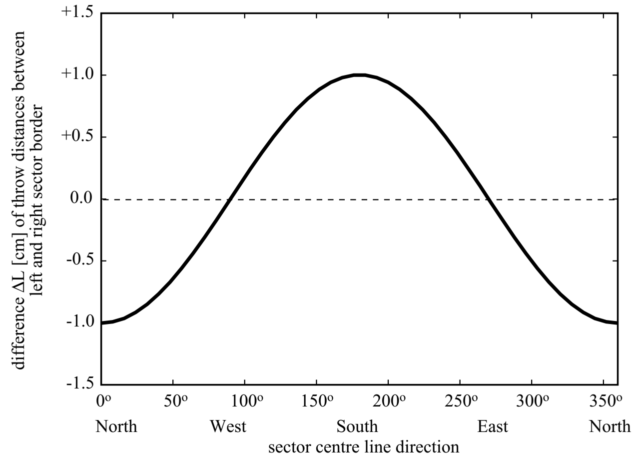

Fig. 5. The difference ΔL between the male world-record hammer throw distances calculated for the left and right border of 40° throwing sector as a function of the orientation of the sector centre line measured counter-clockwise from North for Athens' latitude $\varphi = 38°$, $T = 20°C$, $p = 101\,325$ Pa, $H = 0$ m, $o = 0$, $W = 0$ m/s.

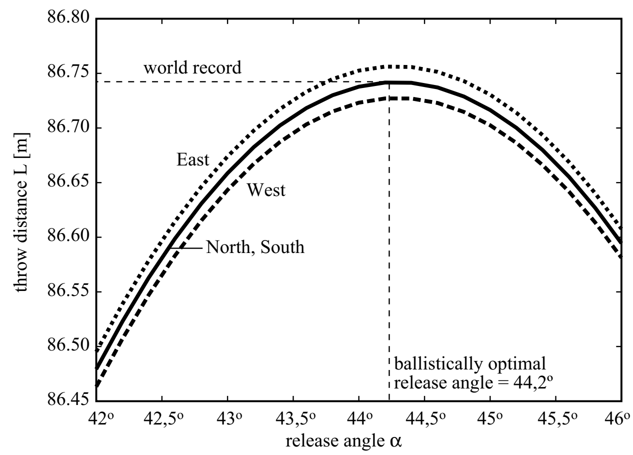

Fig. 6. Male world-record hammer throw distance L as a function of release angle α for four release directions (East, West, North, South) calculated for Athens' latitude $\varphi = 38°$, $T = 20°C$, $p = 101\,325$ Pa, $H = 0$ m, $o = 0$, $W = 0$ m/s. The position of the maxima of the curves belonging to the different release directions is practically the same and it is marked with a vertical broken line corresponding to the value of the ballistically optimal release angle.

3.7. Range versus release angle

Fig. 6 plots the hammer throw distance for Athens ($\varphi = 38°$) as a function of release angle for four release directions. Since the Coriolis force is relatively small, the ballistically optimal release angle (when the range is maximal) is practically independent of the release direction. We can see from Fig. 6 that the ballistically optimal hammer release angle is about 44.2°, which is optimal only if release velocity is independent of release angle. Since no experiments have been carried out to determine the sensitivity of release velocity in the hammer throw, this is probably a reasonable assumption even though the true optimal release angle is likely $<44.2^\circ$. The ballistically optimal release angle is about 42.2° for the shot put. However, probably due to the ability of the thrower to generate higher release speeds at lower angles, shot putters use a nominal release angle of about 37° rather than ca. 42° (Hubbard, 1989). Recently, Hubbard et al. (2001) studied the dependence of release velocity on other release variables in the shot put.

3.8. Correction map of the range versus latitude

Variations in latitude and release direction influence hammer throw and shot put range. In order to compare correctly the hammer throw or shot put ranges L achieved with different release directions β at different latitudes φ, the measured throw distances must be modified in such a way that a term ΔL corresponding to the influence of Earth rotation is added to them. This term, called the "correction term" of range, corrects L for the change of β and φ on the rotating Earth. The correction term $\Delta L(L, \beta, \varphi)$ is positive or negative if the range increases or decreases due to Earth rotation, respectively. Measured throw distances modified by the correction term $L_c = L + \Delta L(L, \beta, \varphi)$ can be considered as the ranges of imaginary hammer throws or shot puts performed with the same reference release direction and at the same reference latitude.

To compute the extrema of the correction term ΔL which can occur on the rotating Earth, as the extrema of the latitude we chose the highest $\varphi_{max} = 67.5^\circ$ (Sodankylä) and lowest $\varphi_{min} = -17.5^\circ$ (Papeete) latitudes at which throwing competitions were ever organized. The northern direction $\beta = 0^\circ$ was set as the reference release direction at the reference latitude $\varphi = 38^\circ$ of Athens. Hence our imaginary "reference stadium" is placed in Athens, where the release direction points northward. The numerical values of the correction term $\Delta L(L, \beta, \varphi)$ were computed for different values of $L_{min} \leqslant L \leqslant L_{max}$ and $0^\circ \leqslant \beta \leqslant 360^\circ$ for a stadium in Sodankylä ($\varphi = 67.5^\circ$) and Papeete ($\varphi = -17.5^\circ$) in the case of the hammer throw (Figs. 7 and 8) and shot put (Figs. 9 and 10).

Figs. 7–10A and 7–10B show the maps of the correction term ΔL of the hammer throw and shot put range L computed for calmness and an east-wind with a speed of 2 m/s (permitted at tail wind by the IAAF for record purposes to sprint races and horizontal jumps). Every point of the maps gives the numerical value of distance ΔL, by which the hammer or shot thrown in Sodankylä or Papeete would fly further or closer in comparison with an imaginary hammer or shot thrown northward in Athens on horizontal ground with the same initial (release) conditions and the same air temperature $T = 20^\circ$C, air pressure $p = 101\,325$ Pa and altitude $H = 0$ m. Figs. 7–10C and 7–10D show the change of ΔL computed for the male world-record range of the hammer throw and shot put as a function of release direction β. Table 6 gives a quick overview of extrema of correction terms in the maps.

In the "correction maps" of range L in Figs. 7–10A the calculated values of ΔL are represented by different grey shades within an annular area on the horizontal throwing ground. In these angular coordinate systems the range is measured radially, and the angle measured from North is the release direction. As a demonstration for using the correction maps in practice, consider the following example: In an athletic competition in Sodankylä a male hammer thrower reached a range of 70 m with an eastward release direction. His trainer (or the spectators, or officials) would like to know how great would be his range in the reference stadium in Athens. One first selects the corresponding correction map (Fig. 7A) then the eastward direction ($\beta = 270^\circ$) and measures a section, corresponding to the range $L =$ 70 m, from the centre of the map. Thereafter the value of the correction term ΔL is read on the map. Finally, adding the correction term to the range $L = 70$ m, the corrected range is obtained.

The correction maps in Figs. 7–10B are asymmetric. One of the reasons for this is the non-linear effect of the wind. As we have seen above (Fig. 3), a head wind with a given speed decreases the range to a greater extent than the enhancement of the throw distance due to a tail wind with the same speed. The second reason for the asymmetry is the influence of the Coriolis force: the ranges are slightly greater eastward than westward.

4. Discussion

4.1. Evolution of hammer throw and shot put world records

The last 10 world records of the male hammer throw and shot put are given in Table 1. On the basis of our results it is evident that the differences ΔL_r of the ranges of the consecutive world records are comparable to, and in many cases smaller than, the values of the correction term ΔL of the world-record ranges (Table 6). Thus in our opinion, it would be time to take into consideration the influence of Earth rotation and other environmental factors (Table 7) on the range of these throwing events. It would be logical to accept the corrected throw distances L_c as the real performances. These corrected ranges can more fairly and physically be compared correctly with each other. Figs. 3, 7–10B, 7–10D and Table 6 demonstrate that a relatively small wind speed of 2 m/s, for instance, has already so great influence on the shot put and hammer throw ranges that it should not be permitted in the approval of the world records.

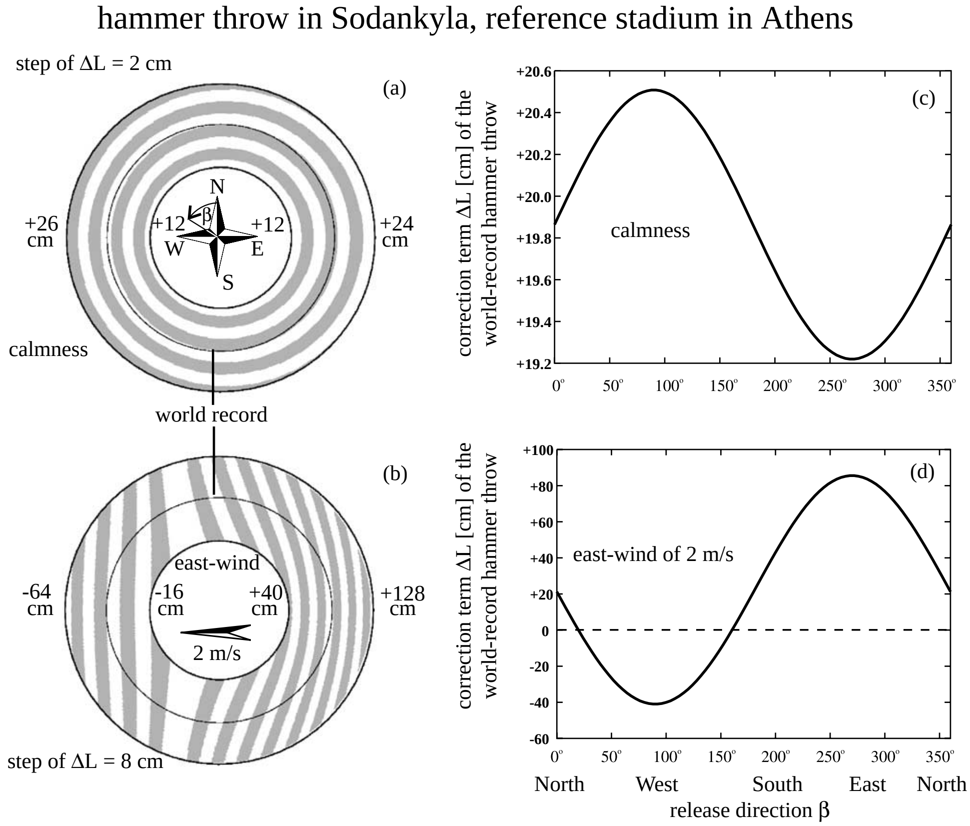

Fig. 7. (A, B) Map of the correction term ΔL of the hammer throw distance L computed for calmness (A) and an east-wind with a speed of 2 m/s (B) for a stadium at the latitude of Sodankylä ($\varphi = 67.5°$) if the reference stadium is at the latitude of Athens ($\varphi = 38°$) where the reference release direction is North ($\beta = 0°$). The border lines of the alternating grey and white zones represent different ΔL values changing between the numerical values given at the margins of the annular maps with a step of $\Delta L = 2$ cm (A) and 8 cm (B). Every point of the map gives the numerical value of distance ΔL, by which the hammer thrown in Sodankylä would fly further or closer in comparison with an imaginary hammer thrown northward in the reference stadium in Athens on horizontal ground with the same initial (release) and environmental conditions $T = 20°$C, $p = 101\,325$ Pa, $H = 0$ m. The inner black circle in the annular maps represents the male world record $L_r = 86.74$ m achieved in 1986. (C, D) Change of ΔL computed for the male world-record range L_r of the hammer throw as a function of release direction β.

4.2. *Comparison of environmental factors influencing range*

We now discuss the influence of environmental factors on the hammer throw and shot put range. Table 7 summarizes the possible maximal change $|\Delta L|_{max}$ of the throw distance L due to variations of different physical and meteorological variables.

Air resistance: In a vacuum the world records of hammer throw ($L = 86.74$ m) and shot put ($L = 23.12$ m) would be 88.09 m ($\Delta L = 135$ cm) and 23.24 m ($\Delta L = 12$ cm), which would mean an enhancement of 2.8% and 0.6%, respectively. Air drag has a considerable effect on hammer throw and shot put ranges.

Wind → air drag: Since air drag depends on the velocity of the thrown implement relative to the air, wind strongly influences the range. A relatively small wind velocity of 2 m/s parallel to the release direction results in a change of the hammer throw and shot put range of about 66 cm (head wind), 62 cm (tail wind) and 4 cm (head wind), 3 cm (tail wind), respectively. Thus, it is pertinent to suppose that the victory of certain hammer throwers and shot putters might have been due to an advantageous wind.

Altitude → gravity + air density → air drag: Altitude H influences range L indirectly (Table 2) through four factors:

(A) *Gravitational acceleration g decreases with altitude*, which has two consequences:

(A1) *The shot or hammer becomes lighter*, thus the thrower can release it with a greater initial velocity. Although reduced gravity increases vertical velocity of the implement at release, it would be difficult to calculate the size of this effect. Thus, we did not take it into consideration, restricting our calculations to throws with the same intial velocity.

(A2) *In a weaker gravitational field at a higher altitude the shot and hammer fly farther*. The range is about $L = v_0^2/g$ for an approximately optimal release angle of 45°. The relative change $\Delta L/L$ of the range due to a relative change $\Delta g/g$ of the gravitational acceleration

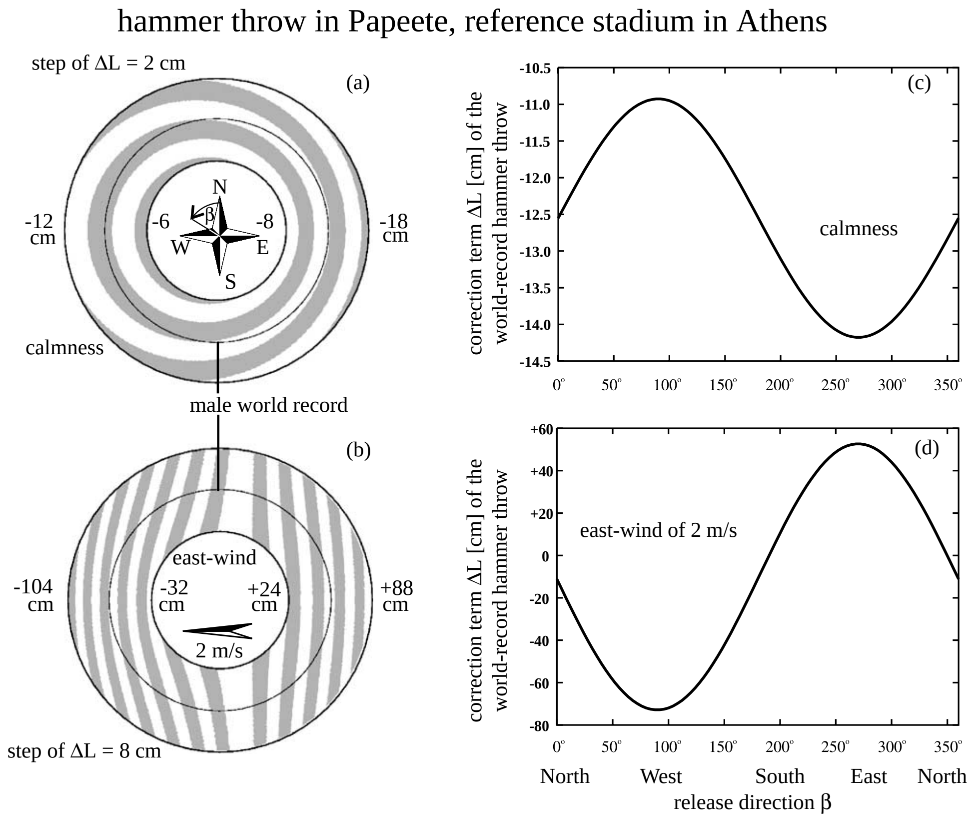

Fig. 8. As Fig. 7 if the stadium is at the latitude of Papeete ($\varphi = -17.5°$).

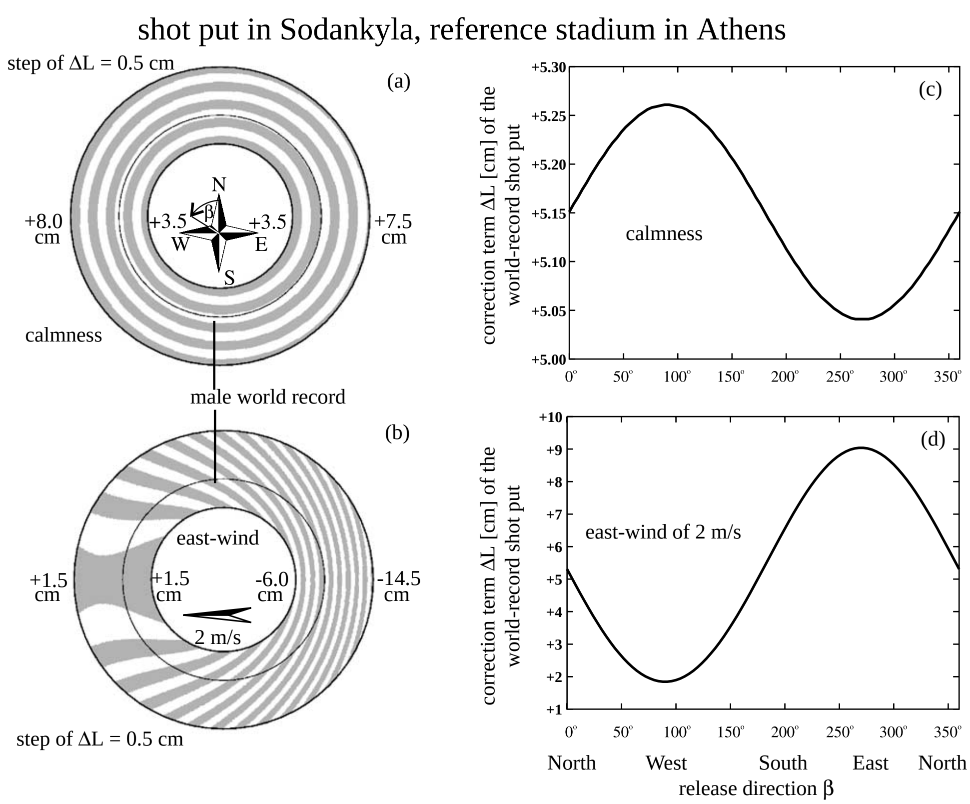

Fig. 9. As Fig. 7 for shot put.

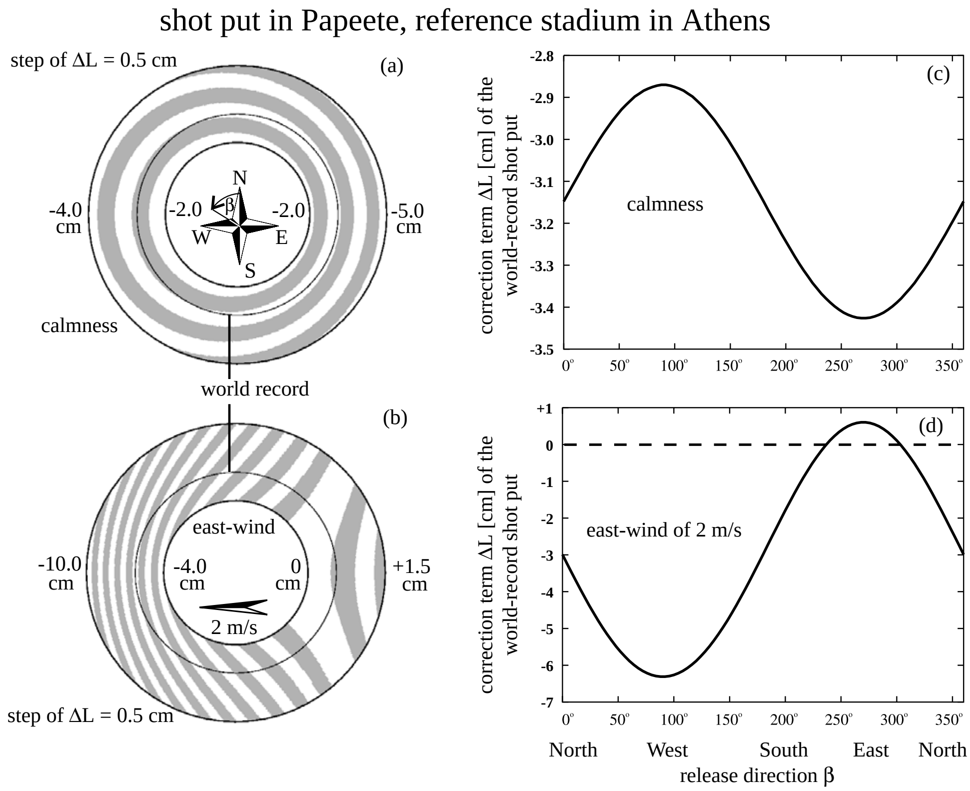

Fig. 10. As Fig. 8 for shot put.

Table 6
The extrema between which the correction term ΔL of the world-record range L of the male hammer throw (Figs. 7, 8) and shot put (Figs. 9, 10) varies computed for a stadium at the latitude of Sodankylä ($\varphi = 67.5°$) or Papeete ($\varphi = -17.5°$) if the reference stadium is in Athens ($\varphi = 38°$) where the reference release direction is North ($\beta = 0°$), and there is calmness or an east-wind with a speed of $W = 2$ m/s

	Male hammer throw		Male shot put	
	Papeete	Sodankylä	Papeete	Sodankylä
Calmness	$-14.1 \leqslant \Delta L \leqslant -10.9$ cm	$+19.2 \leqslant \Delta L \leqslant +20.5$ cm	$-3.4 \leqslant \Delta L \leqslant -2.9$ cm	$+5.05 \leqslant \Delta L \leqslant +5.25$ cm
East-wind	$-72.5 \leqslant \Delta L \leqslant +53.0$ cm	$-40.7 \leqslant \Delta L \leqslant +85.8$ cm	$-6.3 \leqslant \Delta L \leqslant +0.6$ cm	$+1.9 \leqslant \Delta L \leqslant +9.0$ cm

is therefore:

$$\frac{\Delta L}{L} = -\frac{v_0^2}{g^2 L}\Delta g = -\frac{\Delta g}{g} \approx -\frac{2H}{R_{\text{Earth}}} \quad \text{if } H \ll R_{\text{Earth}}. \qquad (6)$$

(B) *Air pressure p decreases with altitude*. Average air pressure p decreases approximately exponentially with altitude H [see Eq. (2)]. The exact change of p depends strongly on the meteorological conditions, nevertheless the relative change of p can reach about 30% at altitudes of several thousands m. The decrease of p results in a decrease in air density ρ, which decreases the air drag, the consequence of which is an increase in distance (Table 4).

(C) *Air temperature T decreases with altitude*. We excluded air temperature from the calculation of the effect of altitude, because it was kept as a separate factor (Table 5). In other words, our calculations figure out what would happen in a competition at higher altitude if temperature were the same as at the lower altitude. If the temperature at the higher altitude was lower, then that would be taken into account separately as a pure temperature effect.

The hammer and shot ranges increase with altitude predominantly because of the decrease of the air density with decreasing air pressure. The decreasing air temperature and gravitational acceleration on range is much smaller.

Latitude → centrifugal force + earth radius change: Due to the centrifugal acceleration and the increase of the Earth radius from the pole to the equator, the net gravitational acceleration $g(\varphi)$ changes as a function of latitude φ. On the basis of the Cassini formula (Caputo, 1967), the relative change of g between the equator and the poles is about 0.53%. This results in a change of the hammer throw and shot put world records of about 45 and 11 cm, respectively. However, there are no

Table 7
The possible maximal change $|\Delta L|_{max}$ of the throw distance L due to the variation of different environmental factors in order of importance computed for the male world-record hammer throw and shot put

Environmental variable	Change of the variable	Physical factor(s) affecting the range	Male world-record hammer throw ($L = 86.74$ m) $\|\Delta L\|_{max}$	Male world-record shot put ($L = 23.12$ m) $\|\Delta L\|_{max}$
Air density $\rho_0 = 1.23$ kg/m^3	$\Delta\rho = 1.23$ kg/m^3 air → vacuum	Air drag	477 cm	13 cm
Wind velocity $W_0 = 0$ m/s	$\Delta W = \pm 2$ m/s parallel to the release direction h: head wind t: tail wind	Air drag	$66_h + 62_t = 128$ cm	$4_h + 3_t = 7$ cm
Altitude $H_0 = 0$ m	$\Delta H = 1000$ m	Gravity, air density (air drag)	55 cm	2 cm
Latitude φ	$\Delta\varphi = 90°$, poles → equator Eastward throw ($\beta = 270°$)	Centrifugal force + Earth radius change	45 cm	11 cm
Latitude φ	$\Delta\varphi = 67.5° - 17.5° = 50°$, Sodankylä → Papeete eastward throw ($\beta = 270°$)	Centrifugal force + Earth radius change	34 cm	9 cm
Air temperature $T_0 = 20°$C	$\Delta T = \pm 10°$C	Air density (air drag)	$2 \times 17 = 34$ cm	$2 \times 0.5 = 1$ cm
Ground obliquity $o_0 = 0$	$\Delta o_{max} = \pm 0.001$	Gravity	$2 \times 8.7 = 17.4$ cm	$2 \times 2.3 = 4.6$ cm
Air pressure $p_0 = 101\,325$ kPa	$\Delta p = \pm 2$ kPa	Air density (air drag)	$2 \times 8 = 16$ cm	$2 \times 0.2 = 0.4$ cm
Release direction β	$\Delta\beta = 180°$, East → West throw on the Equator ($\varphi = 0°$)	Coriolis force	3.4 cm	0.6 cm
Release direction β	$\Delta\beta = 40°$, left → right border of the sector, northward or southward pointing centre line	Coriolis force	1.5 cm	0.2 cm

world-class competitions outside the latitude range $17.5° < |\varphi| < 67.5°$. The hammer throw and shot put range difference between latitudes of 17.5° and 67.5° is about 34 and 9 cm, respectively.

Air temperature → air density → air drag: Air temperature T influences range (Table 5) indirectly through density [see Eq. (5)]. Usually, T changes by less than a few tens Kelvin; for track and field competitions a range of $15°\text{C} \leqslant T \leqslant 35°\text{C}$ is reasonable. For example, the hammer world record of 86.74 m would increase by about 17 cm, if the throw were performed at 30°C instead of 20°C. The implication is that, unless one travels to a town at different altitude, the effect of temperature variation may be larger than that of pressure variation.

Ground obliquity → gravity: It can easily happen that the throwing ground is not exactly horizontal. Since due to the gravitation the time of flight and the range of the shot or hammer depends on the degree of sinking or rising of the ground (Table 3), ground obliquity must be limited. The maximal obliquity of ± 0.001 permitted by the IAAF is equivalent to a maximal change in level of the ground of ± 2.3 cm and ± 8.7 cm for the shot and hammer world records of 23.12 and 86.74 m, respectively. Since both hit the ground at an angle of about 45°, this means that above such an oblique ground the shot and hammer flies 2.3 and 8.7 cm more or less horizontally. Thus, a shot putter or hammer thrower throwing on a sinking ground (with $o = -0.001$) has an advantage of about $2 \times 2.3 = 4.6$ cm, or $2 \times 8.7 = 17.4$ cm against a shot putter or hammer thrower throwing on a rising ground (with $o = +0.001$).

Air pressure → air density → air drag: Range is influenced by the air pressure (Table 4) indirectly through air density, and thus via the air resistance being proportional to the air density ρ. Air pressure varies about 2 kPa among cities at similar altitudes. A ± 2 kPa pressure change would have an effect in the order of 16 cm on hammer range. Air pressure has an important effect on range only when it varies by a large amount, but this occurs only when the two cities are at very different altitudes. Since the not-altitude-dependent part of the air pressure variation is quite small, it has only a small effect on distance.

Release direction → Coriolis force: The release direction influences range because of the Coriolis force, which is maximal on the equator and zero at the poles (Figs. 2, 4–6, 7–10C). In the case of eastward or westward release directions the range is increased or decreased, respectively, by the Coriolis force in both the northern and southern hemispheres. For northward or southward release directions the Coriolis force does not change the range. Using computer simulation, we established that at the equator the Coriolis force changes the hammer and shot ranges by about 3.4 cm and 6 mm, respectively, which is smaller by about one order of magnitude than the change due to a latitude variation of 90°. Hence, a hammer thrower or shot putter throwing eastward has an advantage of about 3.4 cm or 6 mm, respectively, against throwing westward. Since the range is measured with an accuracy of mm, but is rounded to cm, the effect of the Coriolis force on the shot range can be neglected, but it has a measurable influence on hammer range.

The above results hold for both the northern (positive latitudes) and southern (negative latitudes) hemispheres of the Earth. Although the horizontal component of the Coriolis force has different signs at positive and negative latitudes (resulting in clockwise or counter-clockwise slight deviations of the thrown hammer and shot on the northern and southern hemisphere, respectively), the influence of Earth rotation on range at a given positive latitude is practically the same as that at the same negative latitude, because it is dominated by the centrifugal force (being independent of the latitude sign) rather than by the much weaker Coriolis force.

4.3. Normalization of world-record ranges

On the basis of the results of the computer simulations presented in this work we can establish the following: If the influences of environmental factors (in order of importance: wind, altitude, latitude, air temperature, ground obliquity, air pressure and release direction) on throw distance are not taken into consideration in the approval of the new world records, the hammer and shot championship tables may incorrectly represent the relative performances. Ranges achieved by different hammer throwers or shot putters with different release directions in stadia at different latitudes, altitudes, with different wind speeds, air pressures and temperatures cannot be directly compared with each other. The direct comparison done until now is physically incorrect, because throwers or putters may possess an unfair advantage of several decimetres or centimetres, respectively, against other athletes throwing in more disadvantageous conditions.

In principle it is imaginable that after taking into account the influence of environmental factors on range, the hammer throw and shot put championship tables would change. Since meteorological conditions during earlier athletic competitions are usually unknown, the most one could reconstruct subsequently—on the basis of the latitudes of the stadia and the orientations of the throwing sectors—is the extent to which the Earth rotation helped or hindered the throwers.

For example, the advantageous or disadvantageous influence of latitude on throw distance could be eliminated in the future only in that case, if the major international athletic competitions would be organized in stadia placed always at the same latitude in such a way that the orientation of the centre line of the throwing sectors would also be the same. Since in all likelihood this rigorous rule cannot be introduced by the IAAF due to political, diplomatic and technical reasons, we suggest the following, perhaps more acceptable, procedure for a fair and physically correct approval of world records in the hammer throw and shot put.

Using computer simulations, maps of the correction terms for the hammer throw and shot put (Figs. 7–10) can be determined as a function of latitude. On the basis of these correction maps, range achieved at a given latitude with a given release direction can be corrected in such a way as if the throw had been performed in a reference stadium (e.g. Athens) with a reference release direction (e.g. northward). After the throw distance is corrected for latitude and release direction in this way, it can be corrected for other relevant environmental factors such as wind, altitude, ground obliquity, air pressure and temperature as if the throw had been performed under standard physical and meteorological conditions: at sea level ($H_0 = 0$ m), on a horizontal throwing ground ($o_0 = 0$), at normal air pressure ($p_0 = 101\,325$ Pa), at normal air temperature ($T_0 = 20°$C) and under wind-less conditions ($W_0 = 0$ m/s). The influence of wind, altitude, ground obliquity, air pressure and temperature can be characterized by relatively simple formulae (Fig. 3, Tables 2–5). All these corrections could be automatically performed on-line by a computer in a competition.

It would be logical to accept these corrected throw distances as the real performances of the throwers, and these corrected ranges can fairly and correctly be compared with each other. Of course, the prerequisite for such corrections is that wind speed, latitude, orientation of the sector centre line, altitude, ground obliquity, and air pressure and temperature are measured during the throwing events. Nowadays this can easily be performed with standard physical and meteorological measuring instruments.

To our knowledge, the IAAF has never allowed any correction to any result in any event. The most that it has done is to declare illegal for record purposes the times of sprint races and lengths of horizontal jumps made with excessive tail wind (above 2 m/s). They do not adjust the times of these races or the distances of these horizontal jumps; they simply throw them away. They allow quite a bit of environmental variability with no restrictions. For instance, they do not worry about altitude effects for any events, and they do not take into account the influence of favourable winds in the discuss or javelin throw, which can have tremendous effects on performance.

Due to all this, we do not hope that the IAAF will adopt our recommendations for the adjustment of records, since this probably will not happen. Instead, in this work we have focussed on the fact that certain environmental factors affect substantially hammer throw and shot put results. The readers and the officials of the IAAF should decide what to do with this information.

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